

# SYLLABUS MOAO MATHS PART-I

## DETAILS OF SYLLABI

M.A./M.Sc. Mathematics (Previous)

4176  
9-150

### ~~CH-149 (I)~~ Real Analysis

(Duration: Two Semesters / One Year)

Definition and existence of Riemann-Stieltjes integral, Properties of the Integral, Integration and differentiation, the fundamental theorem of Calculus.

Sequences and series of functions, point wise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and differentiation, Weierstrass approximation theorem, Power series, uniqueness theorem for power series, Abel's and Tauber's theorems.

Functions of several variables, linear transformations. Chain rule, Partial derivatives, interchange of the order of differentiation, Derivatives of higher orders, Taylor's theorem.

Lebesgue outer measure. Measurable sets. Measurable functions. Borel and Lebesgue measurability. Non-measurable sets.

The Four derivatives. Functions of Bounded variation. Lebesgue Differentiation Theorem. Differentiation and Integration.

The  $L^p$ -spaces. Convex function, Jensen's inequality. Holder and Minkowski inequalities. Completeness of  $L^p$ , Convergence in Measure, Almost uniform convergence.

#### RECOMMENDED TEXT BOOKS

1. Walter Rudin, Principles of Mathematical Analysis (3<sup>rd</sup> Edition) McGraw-Hill, Kogakusha, 1976, International student edition.
2. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
3. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited Published, New Delhi, 1986 (Reprint 2000).
4. J. H. Williamson, Lebesgue's Integration, Holt, Rinehart and Winston, Inc. New York. 1962.
5. R.G.Bartle, The Elements of Intergration, John Wiley & Sons, Inc. New Youk, 1966.

#### REFERENCES

1. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975
3. A. J. White, Real Analysis; an introduction, Addison-Wesley Publishing Co., Inc., 1968.
4. G. de Barra, Measure Theory and Integration, Wiley Eastern Limited, 1981.
5. I. P. Natanson, Theory of Functions of a Real Variable. Vol. I, Frederick Ungar Publishing Co., 1961.
6. H. L. Royden, Real Analysis, Macmillan Pub. Co. Inc. 4<sup>th</sup> Edition, New York, 1993.
7. Richard L. Wheeden and Antoni Zygmund, Measure and Integral, New An Introduction to Real Analysis, Marcel Dekker Inc. 1977.
8. A. Friedman, Foundations of Modern Analysis, Holt, Rinehart and Winston, Inc., New York, 1970.
9. P.R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
10. T.G.Hawkins, Lebesgue's Theory of Integration: Its Origins and Development, Chelsea, New Your, 1979.
11. K.R.Parthasarathy, Introduction to Probability and Measure, Macmillan Company of India Ltd., Delhi, 1977.
12. Serge Lang, Analysis I & II, Addison-Wesley Publishing Company, Inc. 1969.
13. Inder K. Rana, An Introduction to Measure and Integration, Norosa Publishing House, Delhi, 1997.
14. Walter Rudin, Real & Complex Analysis, Tata McGraw-Hill Publishing Co. Ltd. New Delhi, 1966.

151  
GH-150 (II) Discrete Mathematics

4177  
G-151

(Duration: Two semesters / One year)

**Formal Logic-** Statements. Symbolic Representation and Tautologies. Quantifiers, Predicates and Validity. Propositional logic.

**Semi-groups & Monoids-** Definitions and Examples of Semigroups and Monoids (including those pertaining to concatenation operation). Homomorphism of semigroups and monoids. Congruence relation and Quotient semigroups. Sub-semigroups and submonoids. Direct products. Basic homomorphism Theorem.

**Lattices** – lattices as partially ordered sets. Their properties. Lattices as Algebraic systems. Sublattices. Some special Lattices e.g. Complete, Complemented and Distributive lattices.

**Boolean Algebras-** Boolean Algebras as Lattices. Various Boolean identities. The Switching Algebra examples. Sub-algebras, Direct products and Homomorphisms. Boolean forms and their equivalence. Minterm Boolean Forms, Sum of Products Canonical Forms. Minimization of Boolean Functions. Applications of Boolean Algebra to Switching Theory( using AND, OR & NOT gates).

**Graph Theory** – Definition of (undirected) Graphs, Paths, Circuits, Cycles & Subgraphs. Induced Subgraphs. Degree of a Vertex. Connectivity. Planar Graphs and their properties. Trees Euler's Formula for connected Planar Graphs. Complete & Complete Bipartite Graphs. Kuratowski's Theorem( Statement only) and its use.

Spanning trees, Cut-sets, Fundamental Cut-sets, and Cycles. Minimal Spanning trees and Kruskal's Algorithm. Matrix Representation of Graphs. Euler's theorem on the existence of Eulerian Paths and Circuits. Directed Graphs. In degree and Out degree of a Vertex. Weighted undirected Graphs. Dijkstra's Algorithm.

**Introductory Computability Theory-** Finite State Machines and their Transition Table Diagrams. Equivalence of Finite Set Machines. Reduced Machines. Homomorphism. Finite Automata. Acceptors. Non-deterministic Finite Automata and equivalence of its power to that of Deterministic Finite Automata.

Turing Machine and Partial Recursive Functions. Grammars and Languages-Phrase-Structure Grammars. Language generated by a Grammar. Regular, Context-Free, and Context Sensitive grammars and Languages. Regular sets, Regular Expressions and the Pumping Lemma. Kleene's Theorem.

**RECOMMENDED TEXT BOOKS**

1. J.P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
2. J.L. Gersting, Mathematical Structures for Computer Science, (3<sup>rd</sup> edition), Computer Science Press, New York.
3. Seymour Lipschutz, Finite Mathematics (International edition 1983), McGraw-Hill Book Company, New York.
4. S. Witala, Discrete Mathematics-A Unified Approach, McGraw-Hill Book Co.
5. J.E. Hopcroft and J.D. Ullman, Introduction to Automata Theory, Languages & Computation, Narosa Publishing House.
6. C.L.Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.
7. N.Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India.

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GH-151 (III) Topology

M.A./~~M.Sc.~~ Mathematics (Previous)

4178  
G-149

(Duration: Two Semesters / One Year)

Definition and examples of topological spaces. Closed sets. Closure. Dense subsets. Neighborhoods. Interior, exterior and boundary. Accumulation point and derived sets. Bases and sub-bases, Subspaces and relative topology.

Continuous functions and homeomorphism.

First and Second Countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.

Separation axioms  $T_0, T_1, T_2, T_3, T_4$ ; their Characterizations and basic properties Urysohn's lemma. Tietze extension theorem.

Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification.

Connected spaces. Connectedness on the real line. Components. Locally connected spaces.

Tychonoff product topology in terms of standard sub-base. Connectedness and product spaces. Compactness and product spaces.

RECOMMENDED TEXT BOOKS

1. James R. Munkres, Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. K.D.Joshi, Introduction to General Topology, Wiley Eastern Ltd., 1983.
3. J.L.Kelley, General Topology, Van Nostrand, Reinhold Co., New York, 1995.
4. S. Willard, General Topology, Addison-Wesley, Reading, 1970.
5. K.K. Jha, Advanced General Topology, Nav Bharat Prakashan, Delhi.

REFERENCES:

1. J. Dugundji, Topology, Allyn and Bacon, 1966 (Reprinted in India by Prentice Hall of India Pvt. Ltd.)
2. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.
3. J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1961.
4. L. Steen and J. Seebach, Counter examples in Topology, Holt, Rinehart and Winston, New York, 1970.
5. W. Thron, Topological Structures, Holt, Rinehart and Winston, New York, 1966.
6. N. Bourbaki, General Topology Part I (Transl.), Addison Wesley, Reading, 1966.
7. R. Engelking, General Topology, Polish Scientific Publishers, Warszawa, 1977.
8. W.J.Pervin, Foundations of General Topology, Academic Press Inc. New York, 1964.
9. E.H. Spanier, Algebraic Topology, McGraw-Hill, New York, 1966.
10. Crump W. Baker, Introduction to Topology, Wm C. Brown Publisher, 1991.
11. Sze-Tsen Hu, Elements of General Topology, Holden-Day, Inc. 1965.
12. D. Bushaw, Elements of General Topology, John Wiley & Sons, New York, 1963.
13. M.J. Mansfield, Introduction to Topology, D. Van Nostrand Co. Inc. Princeton, N.J., 1963.
14. B. Mendelson, Introduction to Topology, Allyn & Bacon, Inc., Boston, 1962.
15. C. Berge, Topological Spaces, Macmillan Company, New York, 1963.
16. S.S.Coirms, Introductory Topology, Ronald Press, New York, 1961.
17. Z.P. Mamuzic, Introduction to General Topology, P. Noordhoff Ltd., Groningen, 1963.

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G-152

**GH-152 (IV) Complex Analysis**

(Duration: Two semesters / One year)

Analytic functions :

Complex integration. Cauchy Goursat theorem. Cauchy's Integral formula. Higher order derivatives. Morera's Theorem. Cauchy's inequality. The fundamental theorem of algebra. Taylor's theorem. Laurent's theorem. Meromorphic functions. Zeros and singularities, poles and zeros of meromorphic functions. The argument principle. Rouché's theorem. Inverse function theorem, maximum modulus principle, Theory of residues, Analytic continuation. Residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to  $\arg z$ ,  $\log z$  and  $z^a$ .

Bilinear transformations, their properties and classification. Definitions and examples of Conformal mappings.

Weierstrass factorization theorem. Gamma functions and its properties. Analytic Continuation. Uniqueness of direct analytic continuation. Uniqueness of analytic continuation along a curve. Power series method of analytic continuation Schwarz Reflection principle. Canonical products. Jensen's formula. Poisson-Jensen formula. Hadamard's three circles theorem.

**RECOMMENDED TEXT BOOKS**

1. H.A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.
2. L.V. Ahlfors, Complex Analysis, McGraw-Hill, 1979.
3. S. Lang, Complex Analysis, Addison Wesley, 1977.
4. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
5. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

**REFERENCES**

1. J.B. Conway, Functions of one-Complex variable, Springer-Verlag, International student-Edition, Narosa Publishing House, 1980.
2. Liang-shin Hahn & Bernard Epstein, Classical Complex Analysis, Jones and Brtlett Publishers International, London, 1996.
3. D. Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J. Ablowitz and A.S. Fokas, Complex Variables : Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
5. E. Hille, Analytic Function Theory (2 Vols.), Gonn & Co., 1959.
6. W.H.J. Fuchs, Topics in the Theory of Functions of one Complex Variable, D. Van Nostrand Co., 1967.
7. C. Caratheodory, Theory of Functions (2Vols.), Chelsea Publishing Company, 1964.
8. M. Heins, Complex Function Theory, Academic Press, 1968.
9. S. Saks and A. Zygmund, Analytic Function, Monografie Matematyczne, 1952.
10. E. C. Titchmarsh, The Theory of Functions, Oxford University Press, London.
11. W.A. Veech, A Second Course in Complex Analysis, W.A. Benjamin, 1967.

Optional Paper

~~GH-249 (V)~~ Differential Equations

4180  
G-153

(Duration: Two semesters / One year)

Total differential equation Linear differential equations with variable coefficient including method of variation of parameters.

Preliminaries- Initial value problem,  $n$ th order equations as first order system, concepts of local existence, uniqueness of solutions with examples.

Basic theorems- Ascoli- Arzela Theorem. A theorem on convergence of solutions of family of initial value problems.

Picard-Lindelof theorem – Peano's existence theorem and corollary. Maximal intervals of existence. Extension theorem and corollaries. Kamke's convergence theorem (Statement only) Differential Inequalities and Uniqueness – Gronwall's inequality. Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness Theorems. Successive approximations.

Linear Differential Equations – Linear Systems, Variation of constants, reduction of smaller systems. Basic inequalities, constant coefficients.

Dependence of initial conditions and parameters; Preliminaries. Continuity. Differentiability. Higher Order Differentiability.

RECOMMENDED TEXT BOOKS

1. W.T. Reid, Ordinary Differential Equations, John Wiley & Sons, NY (1971)
2. E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw-Hill, NY (1955).
3. Hartman, Ordinary Differential Equations, John Wiley (1964)

M.A. & M.Sc. Mathematics (Previous)

Optional Paper

~~GH-250 (VI)~~ Differential Geometry

(Duration: Two semesters / One year)

4101  
G-154

Theory of space curves, arc length, tangent and normals, Curvature and torsion of curve given as the intersection of two surfaces, Involute and Evolute.

Metric: The first and second fundamental form, Weingarten equations, Orthogonal trajectories, Mensuier theorem, Gaussian curvature, Euler's theorem, Dupin's theorem, Rodrigue's theorem, Dupin's indicatrix.

Envelopes, Edge of regression, Ruled surface. Developable surface, Monge's theorem, Conjugate directions.

Asymptotic lines, the fundamental equations of surface theory, Gauss's formulae, Gauss characteristics equations, Mainardi Codazzi equations, Weingarten equations, Bonnet's theorem on parallel surface.

Geodesics, Clairaut's theorem, Gauss Bonnet theorem, conformal mapping and Geodesic mappings, Tissot's theorem, Dini's theorem.

RECOMMENDED TEXT BOOKS:

1. T.J. Willmore: Differential Geometry  
ELBS & Cambridge University Press.
2. C.E. Weatherbur: Differential Geometry of three Dimension  
Vol. I ELBS & Cambridge University Press.
3. B.O. Neill Elementary Differential Geometry, Academic Press, 1966.
4. S. Stenberg Lectures on Differential Geometry of curves and surfaces, Prentice-Hall, 1964.
5. M.DoCarmo Differential Geometry of curves and surfaces, Prentice-Hall, 1976.

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G-155

Optional Paper

~~CH-251~~ (VII) Mechanics

G-155

(Duration: Two semesters/One year)

Motion about a fixed axis. The compound Pendulum, Centre of Percussion conservation of Momentum and Energy for finite and impulsive forces. Attraction and potential of rod, disc.

Generalized coordinates. Holonomic and Non-holonomic systems. Scleronomic and Rheonomic systems. Generalized potential. Lagrange's equations of first kind Lagrange's equations of second kind Uniqueness of solution. Energy equation for conservative fields.

Hamilton's variables. Donkin's theorem. Hamilton canonical equations. Cyclic coordinates. Routh's equations Poisson's Identify. Jacobi-Poisson Theorem. Motivating problems of calculus of variations, Shortest distance. Minimum surface of revolution. Brachistochrone problem. Isoperimetric Geodesic. Fundamental Lemma of calculus of variations. Euler's equation for one dependent function and its generalization to (i) 'n' dependent functions, (ii) higher order derivatives.

RECOMMENDED TEXT BOOKS

1. A.S. Ramsey, Dynamics, Part II, The English Language Book Society and Cambridge University Press, 1972.
2. Goldstein, Classical Mechanics (2<sup>nd</sup> edition), Narosa Publishing House, New Delhi.
3. Narayan Chandra Rana & Pramod Sharad Chandra Joag, Classical Mechanics, Tata McGraw Hill, 1991.
4. S.L. Loney, Dynamics, Macmillan and Company, London.

REFERENCES

1. F. Gantmacher, Lectures in Analytic Mechanics, MIR, Publishers, Moscow, 1975.
2. Gelfand and S.V. Fomin, Calculus of Variations: Prentice Hall.
3. A.S. Ramsey, Newtonian Gravitation, The English Language Book Society and the Cambridge University Press.
4. Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University Press 1998.

**GH-252 (VIII) Advanced Abstract Algebra**

(Duration: Two Semesters / One Year)

4183  
G-156

Canonical Forms-Similarity of linear transformations. Invariant subspaces. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms.

Modules, Sub modules. Quotient modules, Homomorphism and Isomorphism theorems. Cyclic modules. Simple modules. Semi-simple modules. Schuler's Lemma. Free modules.

Field theory-Extension fields. Algebraic and transcendental extensions. Separable and inseparable extensions. Normal extensions. Perfect fields. Finite fields. Primitive elements. Algebraically closed fields. Automorphisms of extensions. Galois extensions. Fundamental theorem of Galois theory. Solution of polynomial equations by radicals.

Commutative rings, sub-rings, Ideals. Prime Ideals and maximal ideals, Ring of fractions. Commutative Noetherian rings.

Fundamental structure theorem for finitely generated modules over a principal ideal domain and its applications to finitely generated abelian groups. Rational canonical form. Generalised Jordan forms over any field.

**RECOMMENDED TEXT BOOKS**

1. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975
2. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (2<sup>nd</sup> Edition). Cambridge University Press, Indian Edition, 1997.
3. S. Lang, Algebra, 3<sup>rd</sup> edition, Addison-wesley, 1993.
4. D. S. Malik, J. N. Mordeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.
5. T. Y. Lam, Lectures on Modules and Rings, GTM Vol. 189, Springer-Verlag, 1999.
- 6.

**REFERENCES**

1. P. M. Cohn, Algebra, Vols. I, II & III, John Wiley & sons, 1982,1989,1991.
2. N. Jacobson, Basic Algebra, Vols. I & II, W. H. Freeman. 1980 (also published by Hindustan Publishing Company).
3. I. S. Luther and I. B. S. Passi, Algebra, Vol. I-Groups, II-Rings, Narosa Publishing House (Vol. I-1996. Vol. II-1999).
4. K. B. Datta, Matrix and Linear Algebra, Prentice Hall of india Pvt. Ltd., New Delhi,2000.
5. S. K. Jain, A. Gunawardena and P. B. Bhattacharya, Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag), 2001.
6. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India, 2000.
7. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
8. I. Stewart, Galois Theory, 2<sup>nd</sup> edition, Chapman and Hall, 1989.
9. J. P. Escofier, Galois theory, GTM Vol. 204, Springer, 2001
10. D. S. Passman, A Course in Ring Theory, Wadsworth and Brooks/Cole Advanced Books and Softwares, Pacific Groves, California, 1991.



M.A./M.Sc. Mathematics (Previous)

Optional Paper

GE-253 (IX) THEORY OF NUMBERS

(Duration: Two Semesters / One Year)

4102  
5157

The division algorithm, The Euclidean algorithm, Diophantine equation, The fundamental theorem of Arithmetic, The sieve of Eratosthenes,

Theory of Congruences – Basic properties of congruence, Special divisibility tests, Linear congruencies, Fermat's factorization method, The Little theorem, Wilson's theorem.

Number- Theoretic functions – The functions  $\tau$  and  $\sigma$ , The Mobius Inversion formula, Greatest integer function, Euler's Phi function, Euler's theorem, Properties of the Phi-function.

The order of an integer modulo  $n$ , Primitive roots for primes, Composite numbers having primitive roots, Theory of indices, Euler's criterion, Legendre symbol and its properties, Quadratic reciprocity, Quadratic congruencies with composite moduli.

Perfect number – Mersenne primes, Fermat numbers, Representation of integers as sum of two squares and sum of more than two squares.

RECOMMENDED TEXT BOOKS

1. David M. Burton : Elementary Number Theory  
Universal Book Stall, New Delhi
2. George E. Andrews : Number Theory  
Hindustan Publishing Corporation, New Delhi
3. S. G. Telang : Number Theory  
Tata Mc-Graw Hill Publishing Company Ltd., New Delhi

Optional Paper

M.A./M.Sc. Mathematics (Previous)

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~~GH-254 (X)~~ **MATHEMATICAL STATISTICS**

(Duration: Two Semesters / One Year)

Axiomatic approach to probability, conditional probability, independence of events, Baye's formula, Random variables (discrete and continuous) mathematical expectation and moments.

Bivariate normal, Gamma and Beta distributions, Sampling distributions for mean and variance,  $\chi^2$  (chi-square), t and F distributions.

Criteria for good estimator i.e. unbiasedness, consistency, efficiency and sufficiency. Method of moments, method of least squares and method of maximum likelihood and their application for obtaining estimation of the parameters in simple cases.

Neyman Pearson lemma for testing a simple null hypothesis against a simple alternative hypothesis in the case of Binomial, Poisson and Normal distributions.

Regression plane, Partial and Multiple Correlation's (for three variates only) Large sample tests for mean and proportion, t - test for the hypothesis when samples are independent and dependent.  $X^2$  (chi-square) test, as test of independent and as test of goodness of fit, F test, z test.

RECOMMENDED TEXT BOOKS

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| 1. Parzen                            | : Modern Probability Theory and its application                  |
| 2. Mood, A.M and Graybill, F.A.      | : Introduction to the Theory of Statistics                       |
| 3. Feller, W                         | : An Introduction to the Probability Theory and its applications |
| 4. Goon, A.M. Gupta and Das Gupta B. | : An outline of Statistical Theory Vol.I                         |